On a $v_n = \frac{1}{\sqrt{n}} \cdot u_n$

On a $2\sqrt{n+1} - 2 \leq u_n \leq 2\sqrt{n} - 1$

$\Rightarrow \frac{2\sqrt{n+1}}{\sqrt{n}} - \frac{2}{\sqrt{n}} \leq \frac{1}{\sqrt{n}} \cdot u_n \leq 2 - \frac{1}{\sqrt{n}}$

$$2\cdot\sqrt{1+\frac{1}{n}} - \frac{2}{\sqrt{n}} \leq v_n \leq 2 - \frac{1}{\sqrt{n}}$$

On a $\lim\limits_{n\to+\infty} 2\sqrt{1+\frac{1}{n}} - \frac{2}{\sqrt{n}} = 2$ ($\frac{1}{n} \to 0$, $\frac{2}{\sqrt{n}} \to 0$)

et $\lim\limits_{n\to+\infty} 2 - \frac{1}{\sqrt{n}} = 2$ ($\frac{1}{\sqrt{n}} \to 0$)

Donc

$$\lim_{n\to+\infty} v_n = 2$$